ADAPTIVE AND IMPLICIT HAAR-WAVELET-BASED TIME-DOMAIN INTEGRAL EQUATION ANALYSIS OF STRAIGHT THIN WIRE SCATTERER

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***Abstract*:** An adaptive, implicit, multiresolution time-domain algorithm is applied to solve the integro-differential equation arising from the canonical straight thin wire scattering problem. The extended Haar wavelet system is adopted for the expansion of the induced current along the wire responding to a Gaussian input. Results are compared from implicit and explicit marching-on-time algorithms using Harr wavelet basis functions and pulse basis functions. The implicit algorithm using Haar wavelet system provides an adaptive solution to the transient problem, thus potentially reducing the requirement of computer memory and CPU time.

1. INTRODUCTION

Time-domain techniques have proven to be useful in electromagnetic analysis in transient applications such as short pulse antenna radiation, high-resolution radar scattering, electromagnetic compatibility and electromagnetic pulse interference. Time-domain (TD) methods have gained more and more attention in computational electromagnetics, as they have merits such as obtaining broadband response with a single analysis and good performance with time-varying or nonlinear systems. Most importantly, as in our transient applications in pulsed power, only the early time response is of interest, time domain methods can effectively be truncated, allowing us to compute solutions only for as long as necessary. We prefer integral equation methods when analyzing our electrically large structures, although we have also attempted differential equation methods. Integral equation (IE) methods need only discretize over a surface rather than a volume, and the reduced number of unknowns makes the computational cost both in time and in storage more economical.

In transient applications, there is still room for improvement. Multiresolution analysis has found wide application recently in both TD and frequency domain (FD) methods. The wavelet-based multiresolution analysis (MRA) has proven to be an effective method in the analysis of electrically large structures. Wavelet based MRA can capture the local as well as the global properties of the solutions efficiently. In the vast literature of MRA applied to electromagnetic scattering problems, various wavelet systems have been utilized [1], [2], [3]. The simplest Haar scaling and wavelet functions have finite temporal/spatial support (though sinc-function-type frequency support), and thus, the algorithms adopting Haar subdomain basis functions have wide flexibility when applied to arbitrary domain and are usually easier to implement than other wavelet systems. The Haar wavelet system is also directly analogous to a traditional pulse basis function approach. Most of the MRA research done in time-domain has been applied to differential equations [1], [2], while most of that done with integral equations has been in frequency-domain [3].

In our pulsed power project, it is preferred to utilize multiresolution analysis based on time-domain integral equation method. Previously we used Haar wavelet basis functions in the derivation of the marching-on-time (MOT) algorithm and we have presented satisfactory results with explicit multiresolution formulation [4]. In this paper, we provide an implicit and fully adaptive multiresolution formulation. After the introduction, we state the problem and the formulation in section 2. Numerical results are presented in section 3, followed by discussions and conclusions in section 4.

2. PROBLEM AND FORMULATION

Consider a straight thin wire scatterer exposed to an input TM electric filed. The thin wire is of diameter of 1 centimeter and length of 2 meters. The transient input electric field is a Gaussian pulse

$$E^i(r,t) = E_0 \frac{4}{T\sqrt{\pi}} e^{-\gamma^2} .$$

The wave equation that governs the problem is:

$$\frac{\partial^2 A_z}{\partial t^2} = c^2 \frac{\partial^2 A_z}{\partial z^2} + \frac{\partial E_z^i}{\partial t}, \text{ for } z\in(-z,z),$$

Where *A* is the magnetic vector potential given in terms of the retarded potential integrals. The extended Haar wavelet basis is used to decompose the unknown induced current. Vector potential A_z at each testing point is computed from the segmented currents at previous time steps and the current time step. The implicit scheme is constructed by approximating the derivatives in equation by central finite differences as follows:

$$\frac{\partial^2 A_z}{\partial t^2} = \frac{2}{t_{m,n+1} - t_{m,n-1}} \left(\frac{A_{m,n+1} - A_{m,n}}{t_{m,n+1} - t_{m,n}} - \frac{A_{m,n} - A_{m,n-1}}{t_{m,n} - t_{m,n-1}} \right)$$

$$\frac{\partial^2 A_z}{\partial z^2} = \theta \frac{2}{z_{m+1,n+1} - z_{m-1,n+1}} \left(\frac{A_{m+1,n+1} - A_{m,n+1}}{z_{m+1,n+1} - z_{m,n+1}} - \frac{A_{m,n+1} - A_{m-1,n+1}}{z_{m,n+1} - z_{m-1,n+1}} \right)$$

$$+ (1-2\theta) \frac{2}{z_{m+1,n} - z_{m-1,n}} \left(\frac{A_{m+1,n} - A_{m,n}}{z_{m+1,n} - z_{m,n}} - \frac{A_{m,n} - A_{m-1,n}}{z_{m,n} - z_{m-1,n}} \right)$$

$$+ \theta \frac{2}{z_{m+1,n-1} - z_{m-1,n-1}} \left(\frac{A_{m+1,n-1} - A_{m,n-1}}{z_{m+1,n-1} - z_{m,n-1}} - \frac{A_{m,n-1} - A_{m-1,n-1}}{z_{m,n-1} - z_{m-1,n-1}} \right),$$

where $0 \le \theta \le 1$. $\theta = 0$ gives explicit scheme, which is not stable. The implicit scheme with $0 < \theta < \frac{1}{4}$ is conditional stable while that with $\frac{1}{4} \le \theta \le 1$ is unconditionally stable. Finally, we have formulation:

$$\alpha_0 C_n = F(\overline{A}_{m,n}, A_{m,n-1}, A_{m,n-2}, E^z_{m,n})$$

At each time step, we need to solve a matrix equation. The matrix is related to the degree of the implicitness.

3. NUMERICAL RESULTS

The solver starts from a reasonable level of resolution (usually determined by the input spectrum) and then adaptively chooses the best sets of basis functions at the following time steps. The basis functions are a Haar-wavelet system, thus providing adaptivity both in frequency (resolution) and in space. The procedure is a marching-on-in-time (MOT) scheme. It can terminate computation automatically based on the response, or whenever as you choose.

The transient current computed by the adaptive implicit scheme is plotted in figure 1. The accuracy is satisfactory compared with that from time domain integral equation method adopting pulse basis functions at the highest resolution in both explicit and implicit integration schemes.

We see from figure 1 that the transient frequency response follows the input Gaussian pulse at early time steps, and are dominated by the resonant frequency of the wire at late time steps. To show the adaptiveness of the scheme, the currents along the wire at three time steps are plotted in figures 2, 3 and 4. From these figures, we see that the scheme adopts, in overall, higher levels of resolution at earlier time steps than later time steps. And most importantly, the resolution levels are tailored to the space with the moving of the pulse-like response.

4. CONCLUSIONS

The abstract presents an adaptive and implicit Haar-wavelet-based time-domain integral equation solver and as an example, applied to the analysis of straight thin wire scatterer. The length of the time step is restricted only by the input spectrum. The solver adaptively choose basis functions tailored both to space and to time. This gives a systematic method of reducing the scaling of cost with frequency, which is very important in transient applications, as explained by Walker [5]. Future work will apply it to arbitrary wires and surface applications.

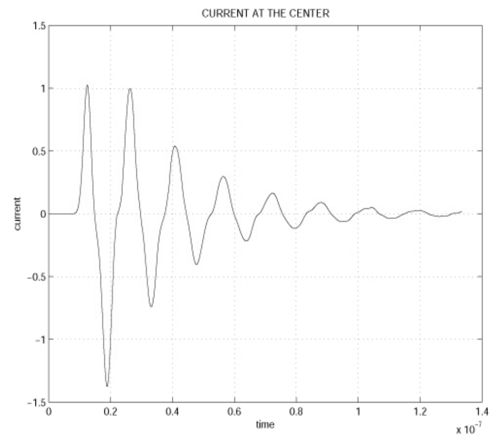

Fig. 1: Current at the center of the wire with time

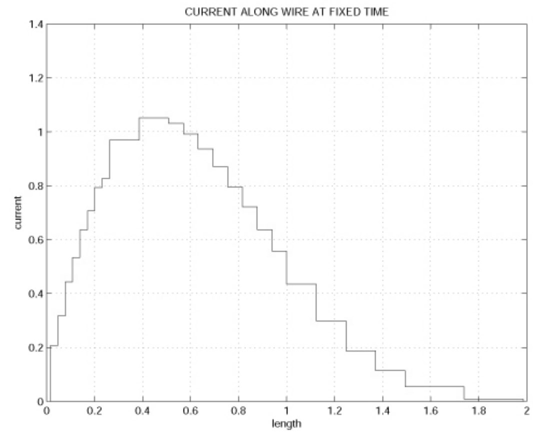

Fig. 2: Current along the wire at the earlier time step

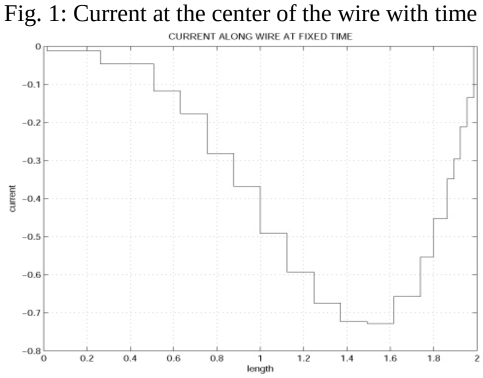

Fig. 3: Current along the wire at the later time step

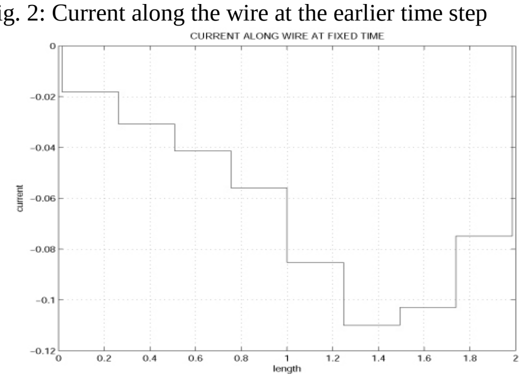

Fig. 4: Current along the wire at very late time

5. ACKNOWLEDGEMENTS

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6. REFERENCES

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