Overview of the LMS Algorithm

The LMS algorithm is an iterative solution to the Wiener–Hopf/normal equations. The standard LMS algorithm can be summarized as:

1. Initialization: $\mathbf{w}[0] = \mathbf{0}$.
2. Filtering: $y[n] = \mathbf{w}^T[n]\tilde{\mathbf{u}}[n]$.
3. Estimation error: $e[n] = d[n] - y[n]$.
4. Tap-weight Update: $\mathbf{w}[n+1] = \mathbf{w}[n] + \mu e[n]\tilde{\mathbf{u}}[n]$.

The tap-weight error follows the recursion:

$$\epsilon[n+1] = (\mathbf{I} - \mu \mathbf{R}_x)\epsilon[n],$$

where $\epsilon[n] = E\{\mathbf{w}[n]\} - \mathbf{w}_{\text{opt}}$. For stability, the step–size factor should be chosen such that :

$$0 < \mu < \frac{2}{\lambda_{\max}},$$

where $\lambda_{\max}$ is the largest eigenvalue of $\mathbf{R}_x$. A small value for the offset factor, e.g., $\beta = 0.01$ is chosen to avoid a divide by zero situation. The decay constant for each mode of the tap–weight error is of the form:

$$\tau_k = \frac{1}{\log(1 - \mu\lambda_k)}, \quad k = 1, 2, \dots, p.$$

The approximate decay constant for each mode of the tap-weight error vector is givven by:

$$\tau_k \approx \frac{1}{\mu\lambda_k}, \quad k = 1, 2, \dots, p.$$

The misadjustment factor M for small step–sizes is approximately:

$$M \approx \frac{1}{2}\mu \ \text{trace}(\mathbf{R}_x).$$

The requirement of a small decay constant τ_k is incompatible with the requirement for a small misadjustment M. They need to be traded-off depending on the signal environment. The normalized version of the LMS algorithm has a tap-weight update of the form:

$$\mathbf{w}[n+1] = \mathbf{w}[n] + \mu e[n]\frac{\tilde{\mathbf{u}}[n]}{||\tilde{\mathbf{u}}[n]||^2 + \beta}.$$

In this case, the step–size factor needs to be chosen such that $0 < \mu < 2$.

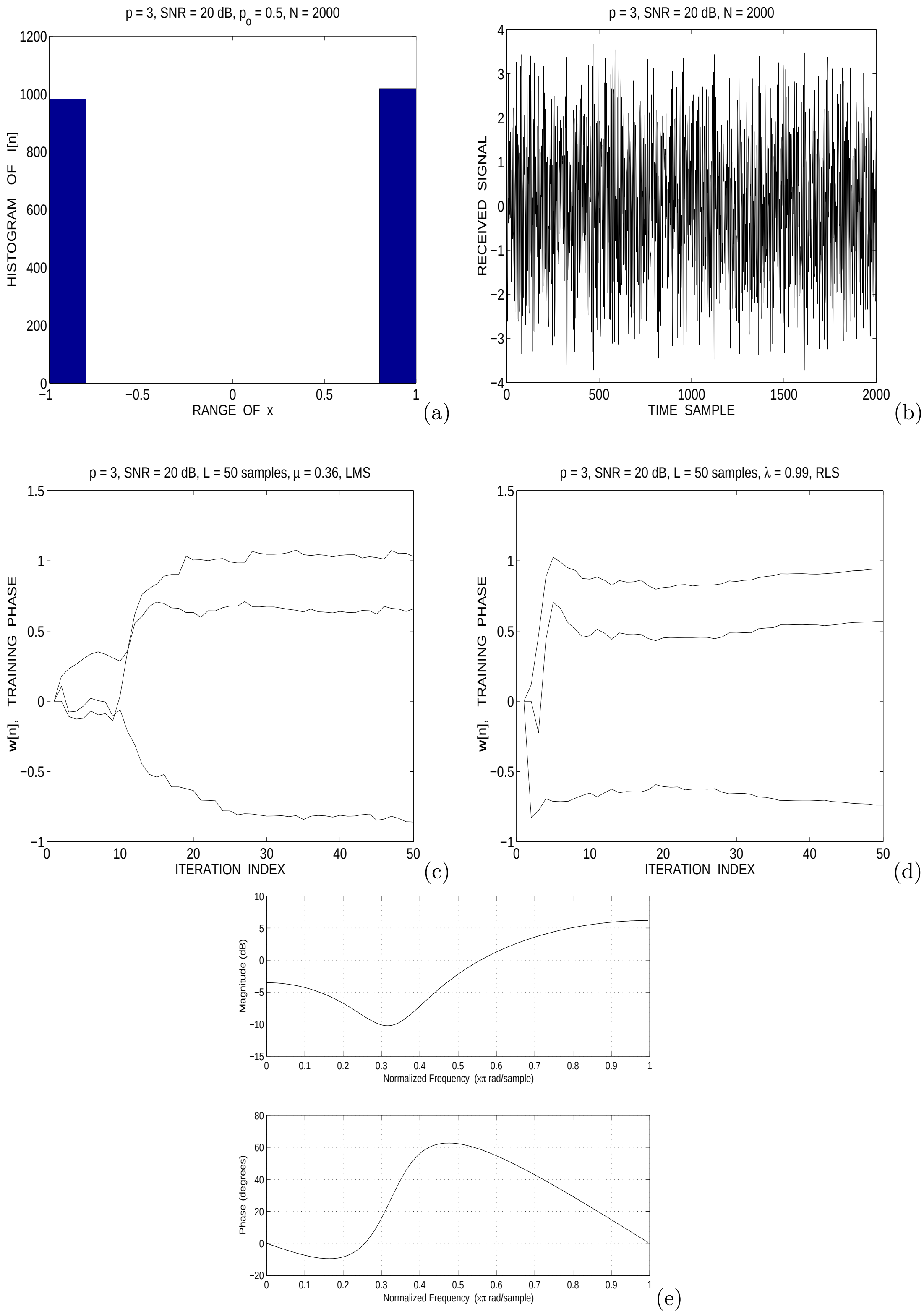

Figure 1: Equalization for a ISI channel: (a) histogram of transmitted signal, (b) sample realization of the received signal,(c,d) filter coefficients of the LMS and RLS algorithms during the training phase. (e) frequency response of the equalizer. The step-size parameter used in the LMS algorithm was $\mu = 0.36$ and the memory factor used in the RLS algorithm was $\lambda = 0.99$. Training was done over the first 50 samples in each case.